universität innsbruck
Department of Mathematics

Design and analysis of favourable time integration methods

Problem class and applications

Consider nonlinear evolution equations of form

$$u'(t) = A(t)\,u(t) + B\big(u(t)\big)\,, \quad t \in (t_0, T)\,.$$

Includes autonomous semi-linear and non-autonomous linear equations.

- Nonlinear Schrödinger equations

  Gross–Pitaevskii equations with rotation (transformed to moving frame, see logo)

- Diffusion-advection-reaction systems

  Deterministic Gray–Scott equations with formation of Turing patterns

  Stochastic Gray–Scott equations driven by fractional Gaussian fields (multiplicative noise)

Main objectives

- Design efficient time integration methods.
- Provide rigorous stability and convergence analysis.

Approach

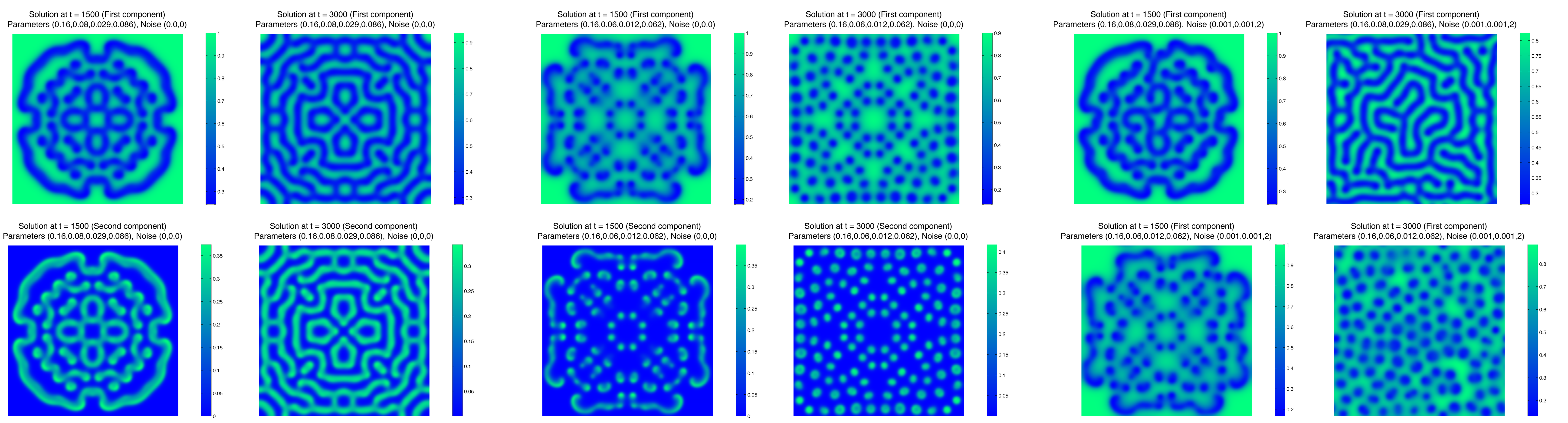
Apply commutator-free quasi-Magnus exponential integrators, i.e., solve sequence of related autonomous equations

$$u'(t) = \mathscr{A}_{jn}\,u(t) + b_j\,B\big(u(t)\big)\,, \qquad t \in (t_n, t_{n+1})\,,$$

$$\mathscr{A}_{jn} = \sum_{k=1}^{K} a_{jk}\,A(t_n + c_k \tau_n)\,, \qquad b_j = \sum_{k=1}^{K} a_{jk}\,,$$

by operator splitting methods ($j \in \{1, \dots, J\}$). In autonomous case, employ local error control with negligible additional cost.

Illustrations (Deterministic / Stochastic Gray–Scott equations)



Movies available at http://techmath.uibk.ac.at/mecht/MyHomepage/Research.html

Collaborators (Design, Convergence analysis, Analysis, SPDEs)

S. Blanes, F. Casas, C. González (Valencia, Castellón, Valladolid)
W. Auzinger, H. Hofstätter, K. Held, O. Koch (Vienna)
B. Kaltenbacher (Klagenfurt)
E. Hausenblas (Leoben)

Mechthild Thalhammer
Department of Mathematics
University of Innsbruck

http://techmath.uibk.ac.at/mecht/