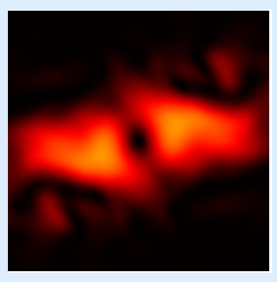

Favourable time integration methods for nonlinear evolution equations

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Situation. Consider nonlinear evolution equation of form

$$u'(t) = A(t)\,u(t) + B\big(u(t)\big)\,, \quad t \in (t_0, T)\,.$$

Special cases. Non-autonomous linear equations, autonomous semi-linear evolution equations.

Areas of application.

- ◇ Nonlinear Schrödinger equations
  - ∘ Gross–Pitaevskii equations (with opening trap)
  - ∘ Gross–Pitaevskii equations with rotation (transformed to moving frame, see logo)
- ◇ Diffusion-advection-reaction systems
  - ∘ Gray–Scott equations with formation of Turing patterns
  - ∘ Stochastic Gray–Scott equations driven by fractional Gaussian fields (multiplicative noise)

Aim. Design and analyse efficient time integration methods.

Approach. Apply commutator-free quasi-Magnus integrators combined with operator splitting methods, that is, solve sequence of related autonomous nonlinear equations

$$u'(t) = \mathscr{A}_{jn}\,u(t) + b_j\,B\big(u(t)\big)\,, \qquad t \in (t_n, t_{n+1})\,,$$
$$\mathscr{A}_{jn} = \sum_{k=1}^{K} a_{jk}\,A(t_n + c_k \tau_n)\,, \qquad b_j = \sum_{k=1}^{K} a_{jk}\,, \qquad j \in \{1,\dots,J\}\,,$$

by means of splitting methods. In autonomous case, employ local error control with negligible additional cost.

Illustration (Deterministic Gray–Scott equations).

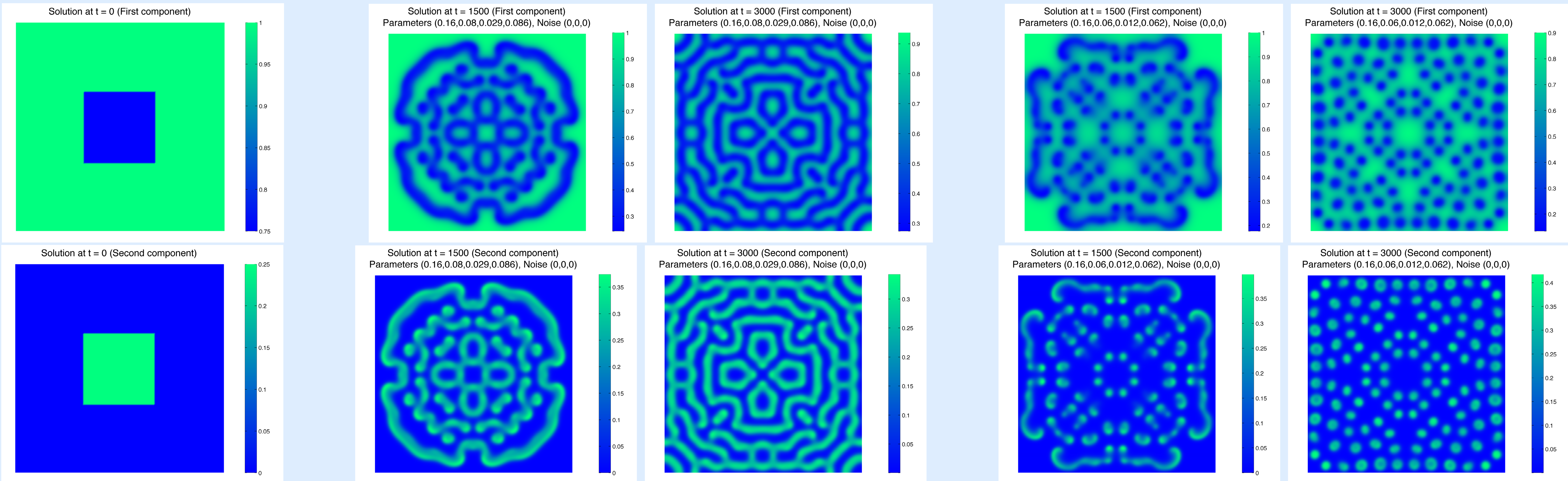
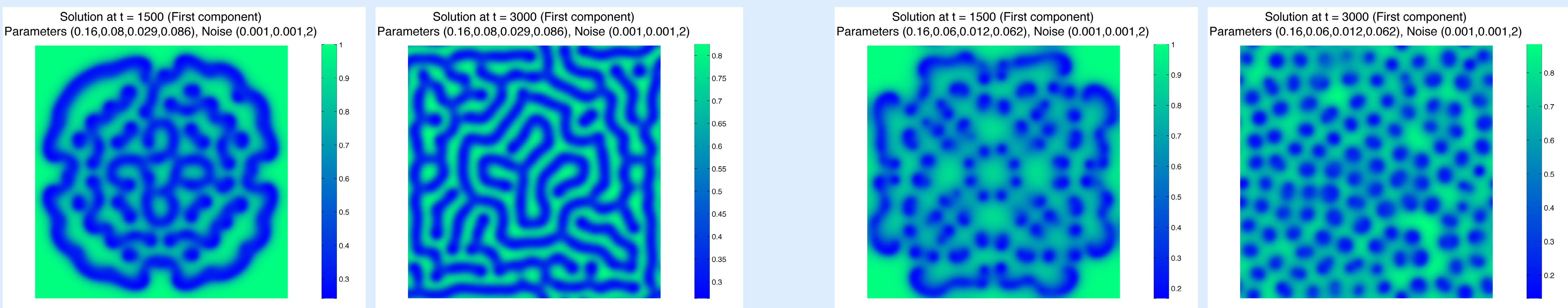


Illustration (Stochastic Gray–Scott equations).



Movies. Available at http://techmath.uibk.ac.at/mecht/MyHomepage/Research.html

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