Projects

The main objectives of the projects are to provide

- connections to applications in atmospheric sciences and
- implementations in MATLAB.

Smaller projects.

1. *Graphical illustrations.*

   Graphical illustrations of functions in single variable and associated tangent lines.

   Graphical illustrations of Taylor series expansions (original function, approximation by Taylor polynomials of different degrees).

   Graphical illustrations of functions in two variables and associated tangent planes.

2. *Finite difference approximations.*

   Implementation of finite difference approximations to first and second derivatives. Computation of approximation errors (for decreasing increments). Observation of effects of round-off errors.

3. *Systems of nonlinear equations.*

   Implementation of Newton's method for one-dimensional case. Graphical illustrations of exact solutions and approximations (in dependence of number of iterations). Computation of approximation errors and observation of quadratic convergence.

   Adaptation to multi-dimensional case.

4. *Euler methods for Dahlquist test equation.*

   Implementation of explicit Euler method for Dahlquist test equation. Graphical illustrations of exact solution values and approximations (for decreasing time increments). Computation of approximation errors and rate of convergence.

   Adaptation to implicit Euler method.

5. *Euler methods for nonlinear ordinary differential equations.*

   Extension of implementation to nonlinear ordinary differential equations.

   Extension of implementation to systems of nonlinear ordinary differential equations.

6. *Second-order time integration methods.*

   Introduction of explicit Runge–Kutta methods for non-autonomous nonlinear systems. Extension of implementation to explicit second-order Runge–Kutta methods.

   Introduction of explicit linear multistep methods for non-autonomous nonlinear systems. Extension of implementation to explicit second-order linear multi-step methods.

7. *Symplectic Euler method for systems of linear ordinary differential equations.*

   Implementation of symplectic Euler method (Lie–Trotter splitting method) for systems comprising two linear ordinary differential equations (non-commutative matrices). Graphical illustrations of exact solution values and approximations (for decreasing time increments). Computation of approximation errors and rate of convergence.

   Extension of implementation to second-order symmetric method (Störmer–Verlet method, Strang splitting method).

Projects.

1. *Fourier series expansions.*

   Implementation of Fourier series expansions (direct summation).
   Efficient implementation by fast Fourier transform.

   Consideration of regular functions. Graphical illustrations of functions and approximations (for increasing number of Fourier basis functions). Computation of approximation errors and observation of spectral accuracy.

   Consideration of functions with discontinuity.

2. *Sine series expansions.*

   Implementation of sine series expansions (direct summation).
   Implementation by odd continuation and application of fast Fourier transform.

   Consideration of localised regular functions: Graphical illustrations of functions and approximations (for increasing number of sine basis functions). Computation of approximation errors and observation of spectral accuracy.

   Consideration of non-localised regular functions.

3. *Nonlinear reaction equations.*

   Connections to applications in atmospheric sciences.

   See lecture on *Mathematische Modellierung: Chemische Reaktionen.*

Numerical solution of nonlinear reaction equation by first- and second-order time integration methods. Observation of exponential decay towards equilibrium.

4. *Linear advection equations.*

   Connections to applications in atmospheric sciences.

   Space and time discretisation of linear advection equations by finite differences and explicit Euler method. Observation of poor stability behaviour.

   Introduction and implementation of upwind-downwind schemes (CFL condition). Computation of approximation errors.

   Introduction and implementation of second-order Lax–Wendroff scheme. Computation of approximation errors.

   See also
   https://en.wikipedia.org/wiki/Upwind_scheme
   https://en.wikipedia.org/wiki/Lax–Wendroff_method

5. *Burgers' equation.*

   Connections to applications in atmospheric sciences.

   Implementation of Newton's method.

   Adaption of implementation for linear advection equations to Burgers' equation.

   Consideration of localised regular initial states and initial states with discontinuity.

6. *Linear diffusion equations.*

   Connections to applications in atmospheric sciences.

   Space and time discretisation of linear diffusion equations subject to homogeneous Dirichlet boundary conditions by central finite differences and explicit Euler method. Observation of poor stability behaviour. Computation of approximation errors.

   Adaptation to implicit Euler method. Observation of favourable stability behaviour.

   Adaptation to Sine spectral method for space discretisation.

7. *Diffusion-advection-reaction equations.*

   Connections to applications in atmospheric sciences.

   Adaptation of implementation to diffusion-advection-reaction equations (splitting, suitable space and time discretisation methods for subproblems).