Kapitel I.3 Mengen, Relationen, Abbildungen

Mengen

Angabe von endlichen Mengen durch Aufzählen der Elemente

Spezialfall der leeren Menge

```
> restart;
```

```
> M0 := { };
  M1 := {a, b, c, d};
  M2 := {Klaus, Bello, Patscherkofel};
  M3 := {1, Pi};
```

$$M0 := \varnothing$$

$$M1 := \{a, b, c, d\}$$

$$M2 := \{Bello, Klaus, Patscherkofel\}$$

$$M3 := \{1, \pi\} \quad \textbf{(1)}$$

Überprüfen der Elementeigenschaft

Vorsicht in MAPLE!

Klein- und Großschreibung (Buchstabe pi versus mathematische Konstante Pi)

```
> evalf(pi);
  evalf(Pi);
```

$$\pi$$

$$3.141592654 \quad \textbf{(2)}$$

```
> evalb(Bello in M1);
  evalb(Bello in M2);
  evalb(pi in M3);
  evalb(Pi in M3);
```

$$false$$

$$true$$

$$false$$

$$true \quad \textbf{(3)}$$

Geänderte Reihenfolge oder Wiederholung spielen keine Rolle

```
> restart;
```

```
> M := {a, b, c};
  N := {c, b, a};
  evalb(M = N);
  N := {a, a, b, c};
  evalb(M = N);
```

$$M := \{a, b, c\}$$

$$N := \{a, b, c\}$$

$$true$$

$$N := \{a, b, c\}$$

$$true \quad \textbf{(4)}$$

Teilmengen

```
> restart;
```

```
> M := {a, b, c};
  N1 := {a, b};
  evalb(N1 subset M);
  N2 := {a, d};
  evalb(N2 subset M);
```

$$M := \{a, b, c\}$$
$$N1 := \{a, b\}$$
$$true$$
$$N2 := \{a, d\}$$
$$false \quad \textbf{(5)}$$

Vereinigung von Mengen (Zusammenhang mit "oder")

```
> restart;
```

```
> M1 := {1, 3, 5};
  M2 := {2, 4, 6};
  M := M1 union M2;
```

$$M1 := \{1, 3, 5\}$$
$$M2 := \{2, 4, 6\}$$
$$M := \{1, 2, 3, 4, 5, 6\} \quad \textbf{(6)}$$

Durchschnitt von Mengen (Zusammenhang mit "und")

```
> restart;
```

```
> M := {1, 2, 3, 4, 5, 6};
  N := M intersect {1, 3, 5};
  N := M intersect {1, 3, 5, 7};
```

$$M := \{1, 2, 3, 4, 5, 6\}$$
$$N := \{1, 3, 5\}$$
$$N := \{1, 3, 5\} \quad \textbf{(7)}$$

Mengendifferenz (Zusammenhang mit "und" sowie "Verneinung")

```
> restart;
```

```
> M := {1, 2, 3, 4, 5, 6};
  N := M minus {2, 4, 6};
  N := M minus {2, 4, 6, 8};
```

$$M := \{1, 2, 3, 4, 5, 6\}$$
$$N := \{1, 3, 5\}$$
$$N := \{1, 3, 5\} \quad \textbf{(8)}$$

Kartesisches Produkt von Mengen
Reihenfolge wesentlich
Vorsicht in MAPLE!
Schreibweise für Paare (Operationen ausführbar, jedoch mit Klammerung leichter lesbar)

```
> restart;
```

```
> A := (a, 1);
  A[1];
  A + A;
  A - (1, a);
```

```
A - [1, a];
A := [a, 1];
A[1];
A + A;
A - [1, a];
```

$$A := a, 1$$
$$a$$
$$2\,a, 2$$
$$a - 1, 1 - a$$
$$(a, 1) + [-1, -a]$$
$$A := [a, 1]$$
$$a$$
$$[2\,a, 2]$$
$$[a - 1, 1 - a] \quad \textbf{(9)}$$

```
> M1 := {a, b};
M1[1];
M1[2];
m1 := numelems(M1);
M2 := {1, 2, 3};
m2 := numelems(M2);
```

$$M1 := \{a, b\}$$
$$a$$
$$b$$
$$m1 := 2$$
$$M2 := \{1, 2, 3\}$$
$$m2 := 3 \quad \textbf{(10)}$$

```
> M := {[a, 1], [b, 1], [a, 2], [b, 2], [a, 3], [b, 3]};
evalb([a, 1] in M);
evalb([1, a] in M);
m1 · m2 = numelems(M);
```

$$M := \{[a, 1], [a, 2], [a, 3], [b, 1], [b, 2], [b, 3]\}$$
$$true$$
$$false$$
$$6 = 6 \quad \textbf{(11)}$$

Alternative

```
> N := { } :
counter := 0 :
for i from 1 to m1 do
  for j from 1 to m2 do
    counter := counter + 1;
    N := N union {[M1[i], M2[j]]};
  od;
```

```
od;
counter = numelems(N);
N;
M;
```

$$6 = 6$$

$$\{[a, 1], [a, 2], [a, 3], [b, 1], [b, 2], [b, 3]\}$$

$$\{[a, 1], [a, 2], [a, 3], [b, 1], [b, 2], [b, 3]\} \quad \textbf{(12)}$$

Mögliche Ergebnisse beim Wurf zweier Münzen bzw. Würfel

```
> restart;
> M := {Kopf, Zahl};
  N := { } :
  counter := 0 :
  for i from 1 to numelems(M) do
    for j from 1 to numelems(M) do
      counter := counter + 1;
      N := N union {[M[i], M[j]]};
    od;
  od;
  counter = numelems(N);
  N;
```

$$M := \{Kopf, Zahl\}$$

$$4 = 4$$

$$\{[Kopf, Kopf], [Kopf, Zahl], [Zahl, Kopf], [Zahl, Zahl]\} \quad \textbf{(13)}$$

```
> restart;
> M := {1, 2, 3, 4, 5, 6};
  N := { } :
  counter := 0 :
  for i from 1 to numelems(M) do
    for j from 1 to numelems(M) do
      counter := counter + 1;
      N := N union {[M[i], M[j]]};
    od;
  od;
  counter = numelems(N);
  N;
```

$$M := \{1, 2, 3, 4, 5, 6\}$$

$$36 = 36$$

$$\{[1, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [2, 1], [2, 2], [2, 3], [2, 4], [2, 5], [2, 6], [3, 1], [3, 2], [3, 3], [3, 4], [3, 5], [3, 6], [4, 1], [4, 2], [4, 3], [4, 4], [4, 5], [4, 6], [5, 1], [5, 2], [5, 3], [5, 4], [5, 5], [5, 6], [6, 1], [6, 2], [6, 3], [6, 4], [6, 5], [6, 6]\} \quad \textbf{(14)}$$

```
> M := {1, 2, 3, 4};
  N := { } :
  counter := 0 :
  for i from 1 to numelems(M) do
```

```
    for j from 1 to numelems(M) do
      for k from 1 to numelems(M) do
        counter := counter + 1;
        N := N union {[M[i], M[j], M[k]]};
      od;
    od;
  od;
  counter = numelems(N);
  N;
```

$$M := \{1, 2, 3, 4\}$$

$$64 = 64$$

$$\{[1,1,1],[1,1,2],[1,1,3],[1,1,4],[1,2,1],[1,2,2],[1,2,3],[1,2,4],[1,3,1],[1,3,2],[1,3,3],[1,3,4],[1,4,1],[1,4,2],[1,4,3],[1,4,4],[2,1,1],[2,1,2],[2,1,3],[2,1,4],[2,2,1],[2,2,2],[2,2,3],[2,2,4],[2,3,1],[2,3,2],[2,3,3],[2,3,4],[2,4,1],[2,4,2],[2,4,3],[2,4,4],[3,1,1],[3,1,2],[3,1,3],[3,1,4],[3,2,1],[3,2,2],[3,2,3],[3,2,4],[3,3,1],[3,3,2],[3,3,3],[3,3,4],[3,4,1],[3,4,2],[3,4,3],[3,4,4],[4,1,1],[4,1,2],[4,1,3],[4,1,4],[4,2,1],[4,2,2],[4,2,3],[4,2,4],[4,3,1],[4,3,2],[4,3,3],[4,3,4],[4,4,1],[4,4,2],[4,4,3],[4,4,4]\} \quad \textbf{(15)}$$

>