Kapitel I.3 Mengen, Relationen, Abbildungen
Funktionen

```
> restart;
> f := proc(x)
    description "definition of a function {a,b,c,d} -> {1,2,3}";
      if x = a or x = b then f(x) := 1 elif x = c then f(x) := 2 elif x = d then f(x) := 3
      end if;
    end proc:
  f(a);
  f(b);
  f(c);
  f(d);
```

1

1

2

3 **(1)**

Vorsicht in MAPLE!
Klammerung für Paare

```
> Graph_f := {[a,f(a)], [b,f(b)], [c,f(c)], [d,f(d)]};
```

$Graph\_f := \{[a, 1], [b, 1], [c, 2], [d, 3]\}$ **(2)**

Vorsicht in MAPLE!
Zwei Varianten möglich.
(Besser gelöst z.B. in Matlab, wo zwischen Funktionsname und Funktionswert unterschieden wird
output = funktionsname(input))

```
> restart;
  f := proc(x)
  description "definition of a function {a,b,c,d} -> {1,2,3}";
    if x = a or x = b then f := 1 elif x = c then f := 2 elif x = d then f := 3
    end if;
  end proc:
  f(a);
  f(b);
  f(c);
  f(d);
  Graph_f := {[a,f(a)], [b,f(b)], [c,f(c)], [d,f(d)]};
```

Warning, `f` is implicitly declared local to procedure `f`
Warning, `f` is implicitly declared local to procedure `f`

1

1

2

3

$Graph\_f := \{[a, 1], [b, 1], [c, 2], [d, 3]\}$ **(3)**

Bildmenge

```
> {f(a),f(b),f(c),f(d)};
```

$$\{1, 2, 3\} \quad (4)$$

```
> M := { };
  for x in {a, c} do
   M := M union {f(x)};
  od;
```

$$M := \varnothing$$

$$M := \{1\}$$

$$M := \{1, 2\} \quad (5)$$

Urbildmengen

```
> for y in {1, 2, 3} do
  Urbild[y] := { } :
  for x in {a, b, c, d} do
   if f(x) = y then Urbild[y] := Urbild[y] union {x} end if:
  od:
  print(y, Urbild[y]);
  od:
  Urbild[1, 3] := Urbild[1] union Urbild[3];
```

$$1, \{a, b\}$$

$$2, \{c\}$$

$$3, \{d\}$$

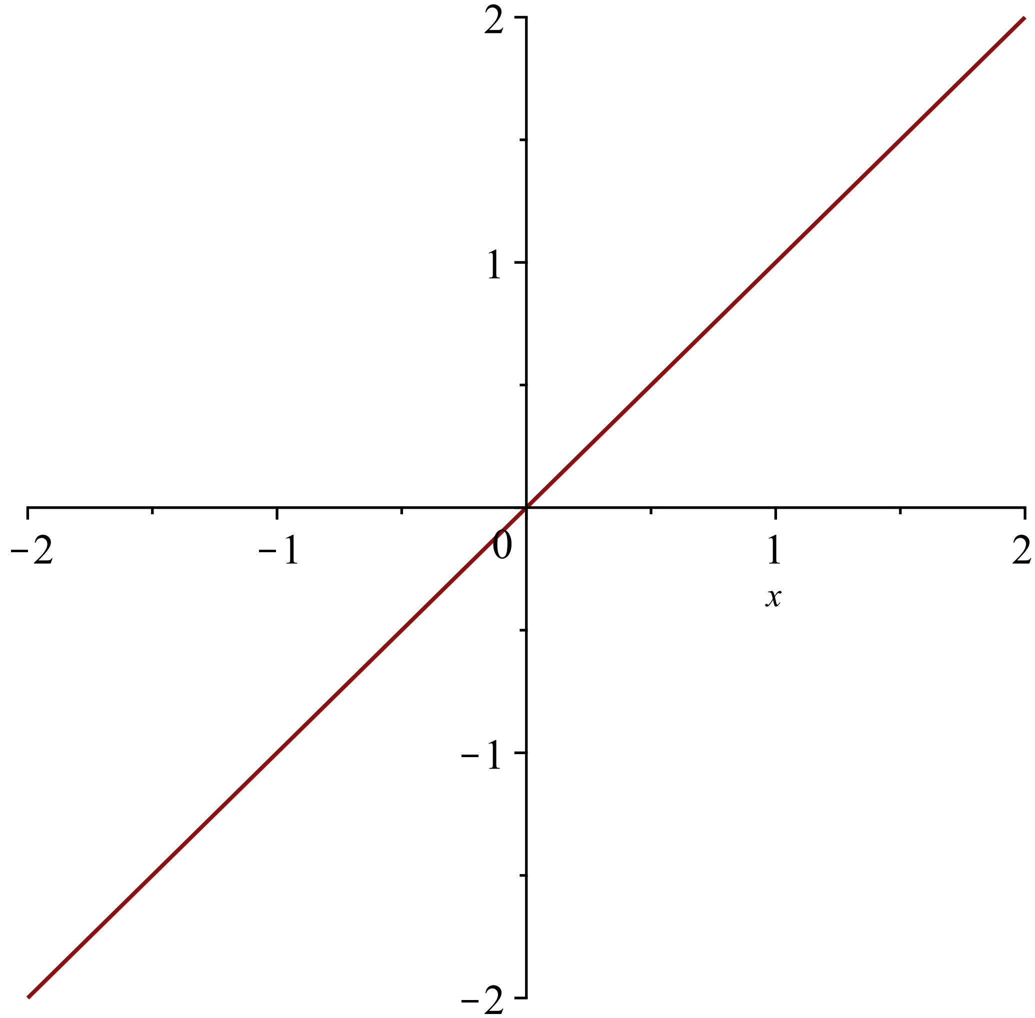
$$Urbild_{1, 3} := \{a, b, d\} \quad (6)$$

Identitätsfunktion auf Teilbereich der reellen Zahlen

```
> restart;
```

```
> id := x -> x;
  plot(id(x), x = -2 .. 2);
```

$$id := x \mapsto x$$

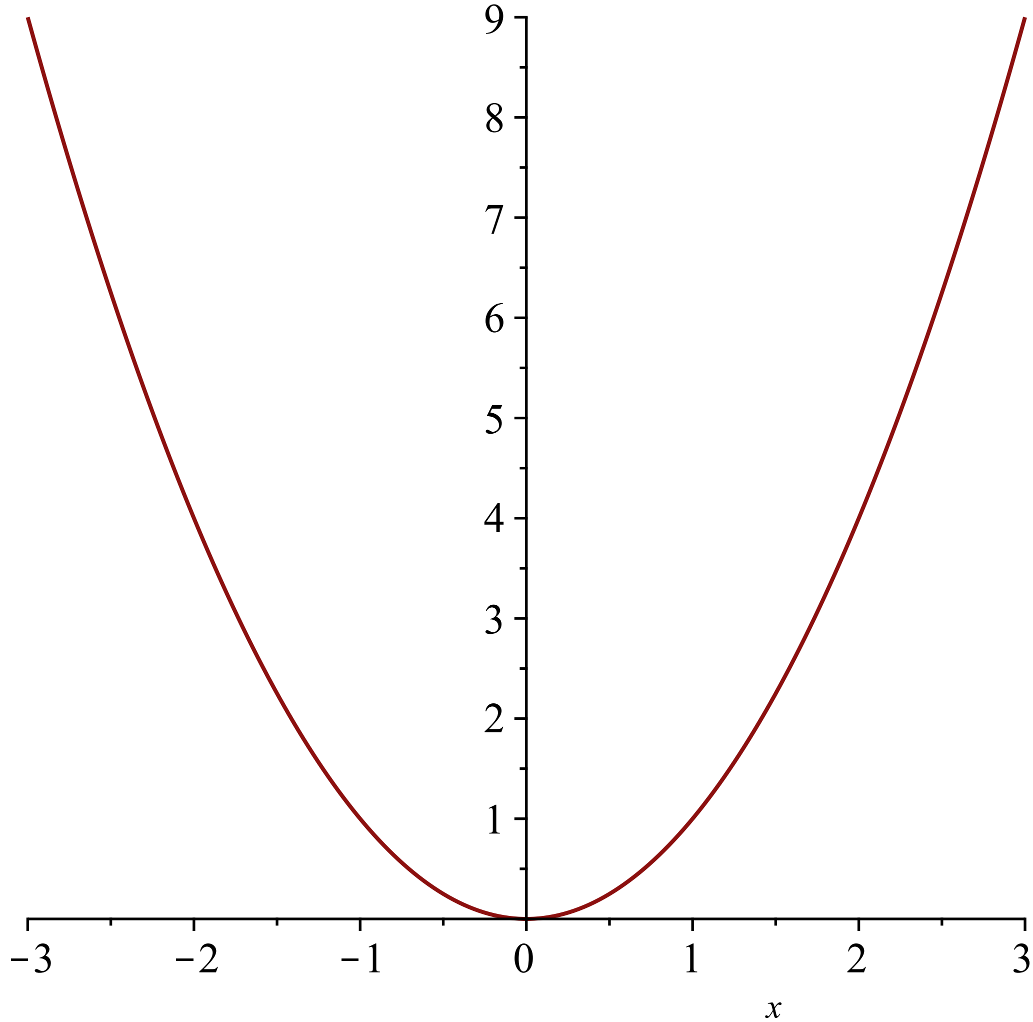


Quadratische Funktion (Graph ist Parabel)
Einschränkung auf nicht-negative reelle Zahlen
Wurzelfunktion als zugehörige inverse Funktion
"Spiegelung" an Identität

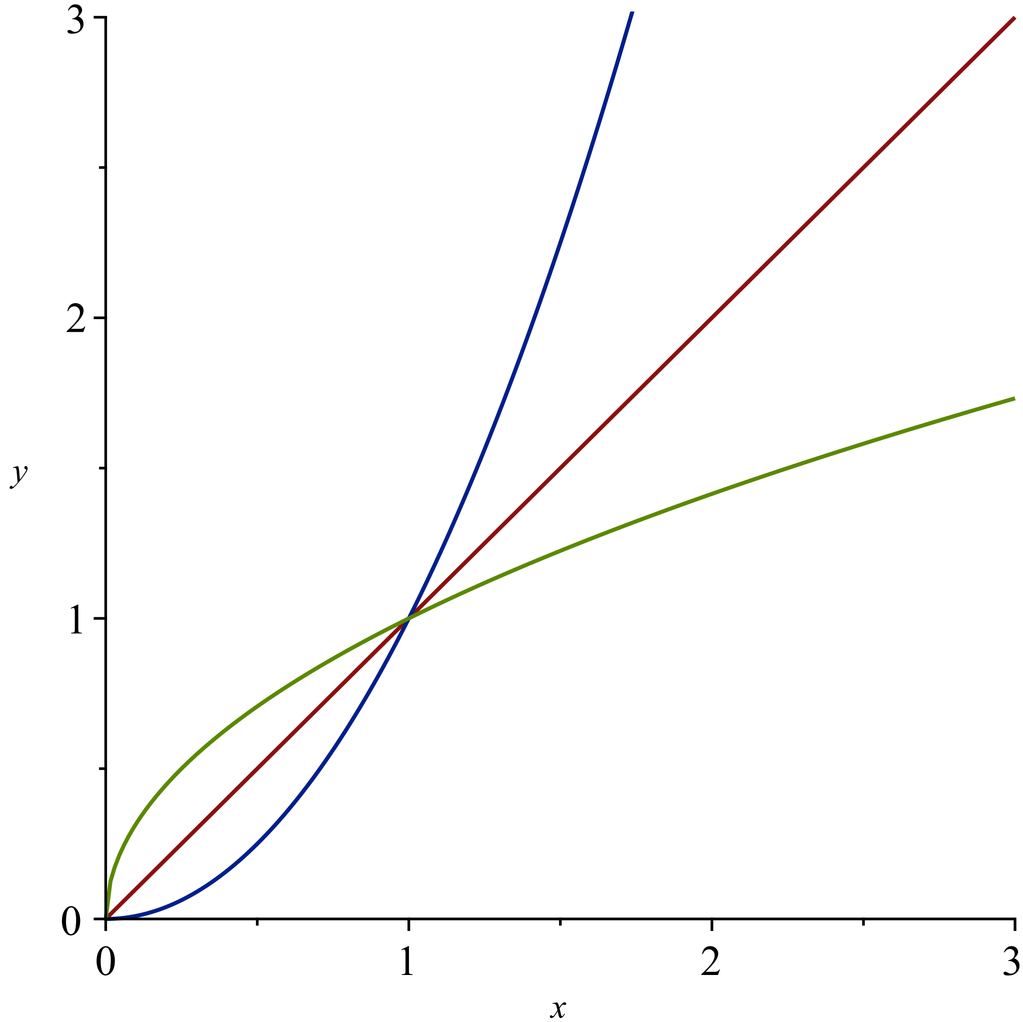
> *restart*;

> $f := x \rightarrow x^2;$
$plot(f(x), x = -3 .. 3);$

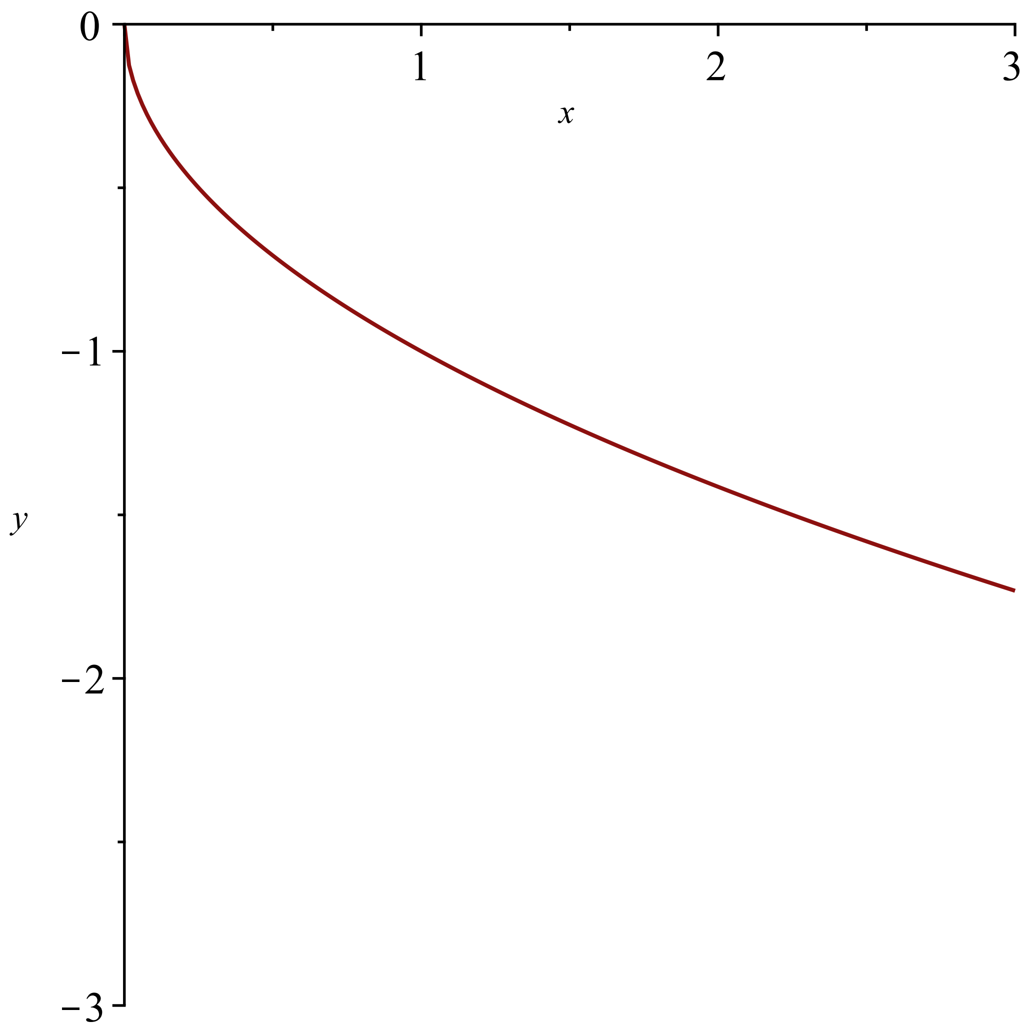
$$f := x \mapsto x^2$$



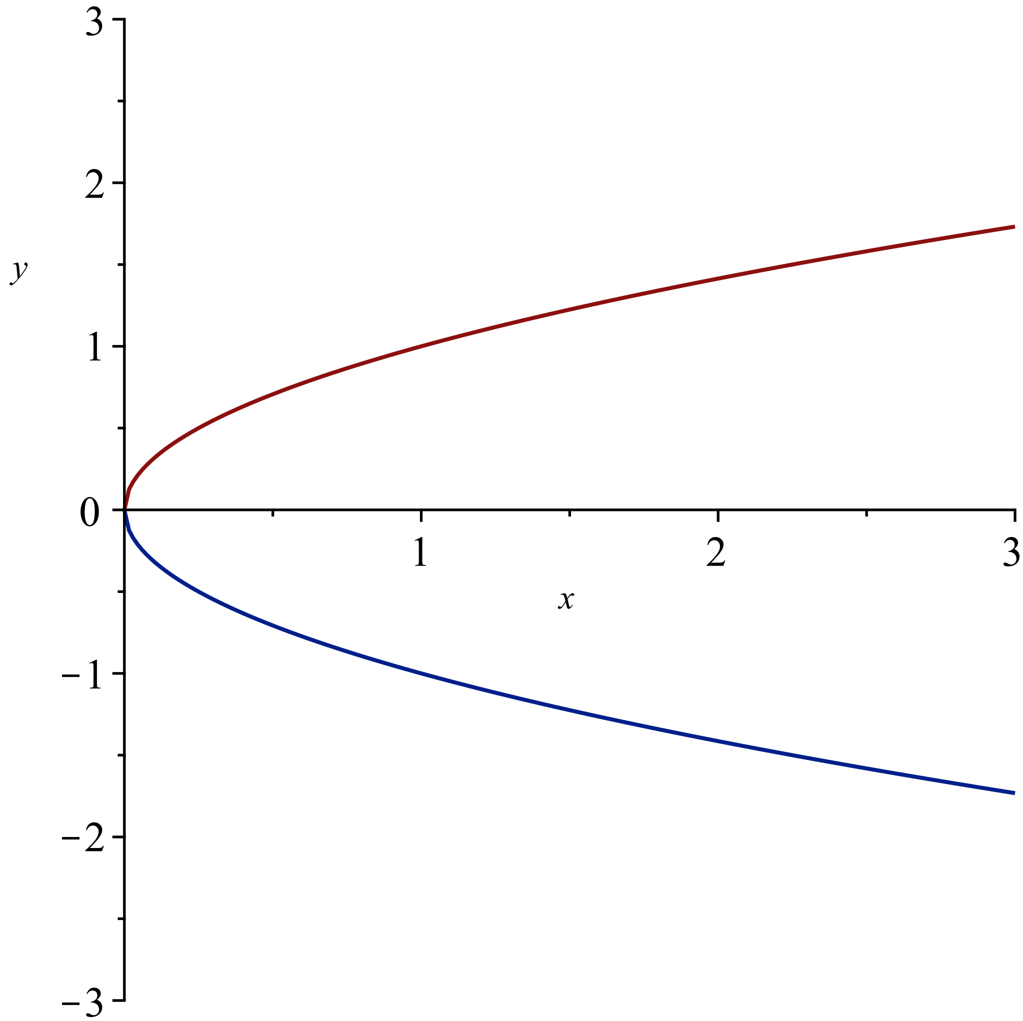
> $plot(\{x, f(x), \mathrm{sqrt}(x)\}, x = 0 .. 3, y = 0 .. 3);$



> $plot(-\operatorname{sqrt}(x), x = 0 .. 3, y = -3 .. 0);$
$plot(\{\operatorname{sqrt}(x), -\operatorname{sqrt}(x)\}, x = 0 .. 3, y = -3 .. 3);$



0
1
2
3
x
y
−1
−2
−3



Komposition

I.a. nicht kommutativ, d.h. f(g(x)) verschieden von g(f(x))

> $restart;$

> $f := x \rightarrow x^2;$

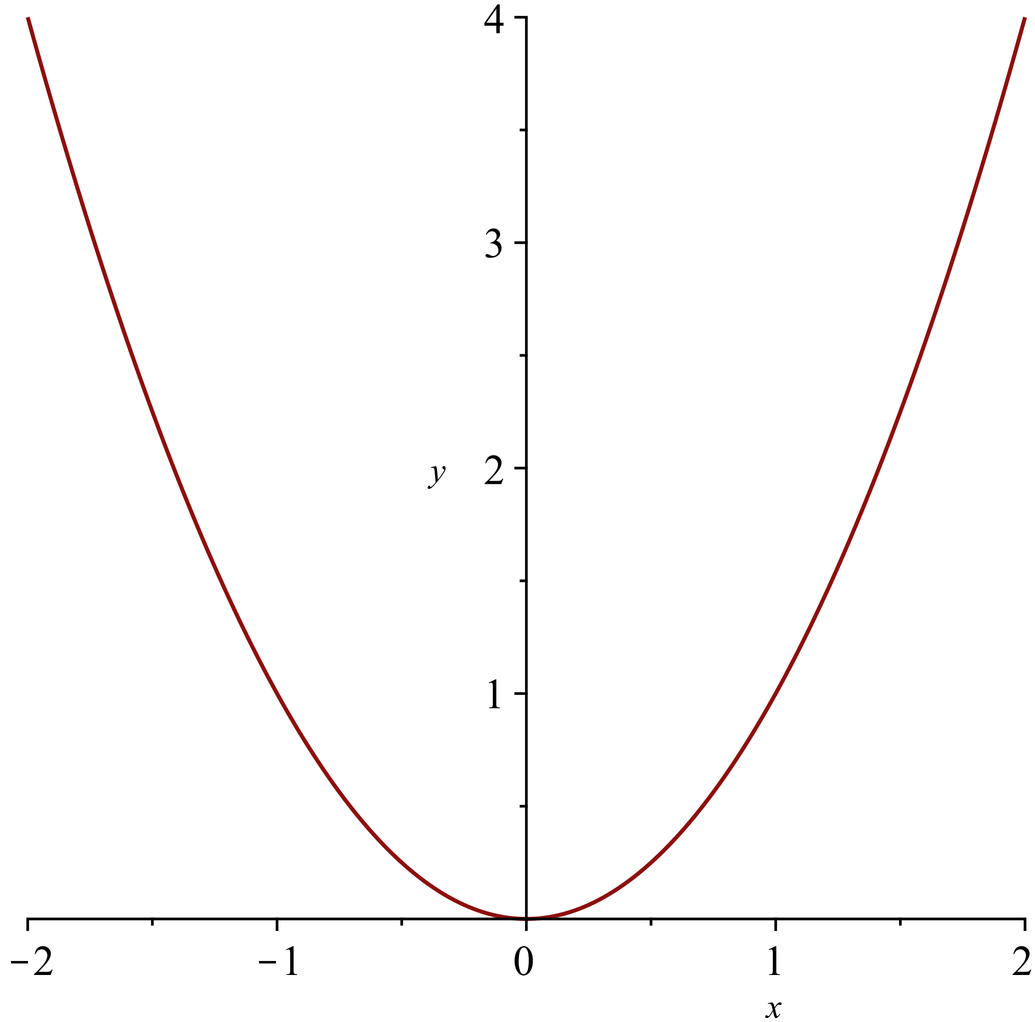
$g := x \rightarrow x + 1;$

$$f := x \mapsto x^2$$

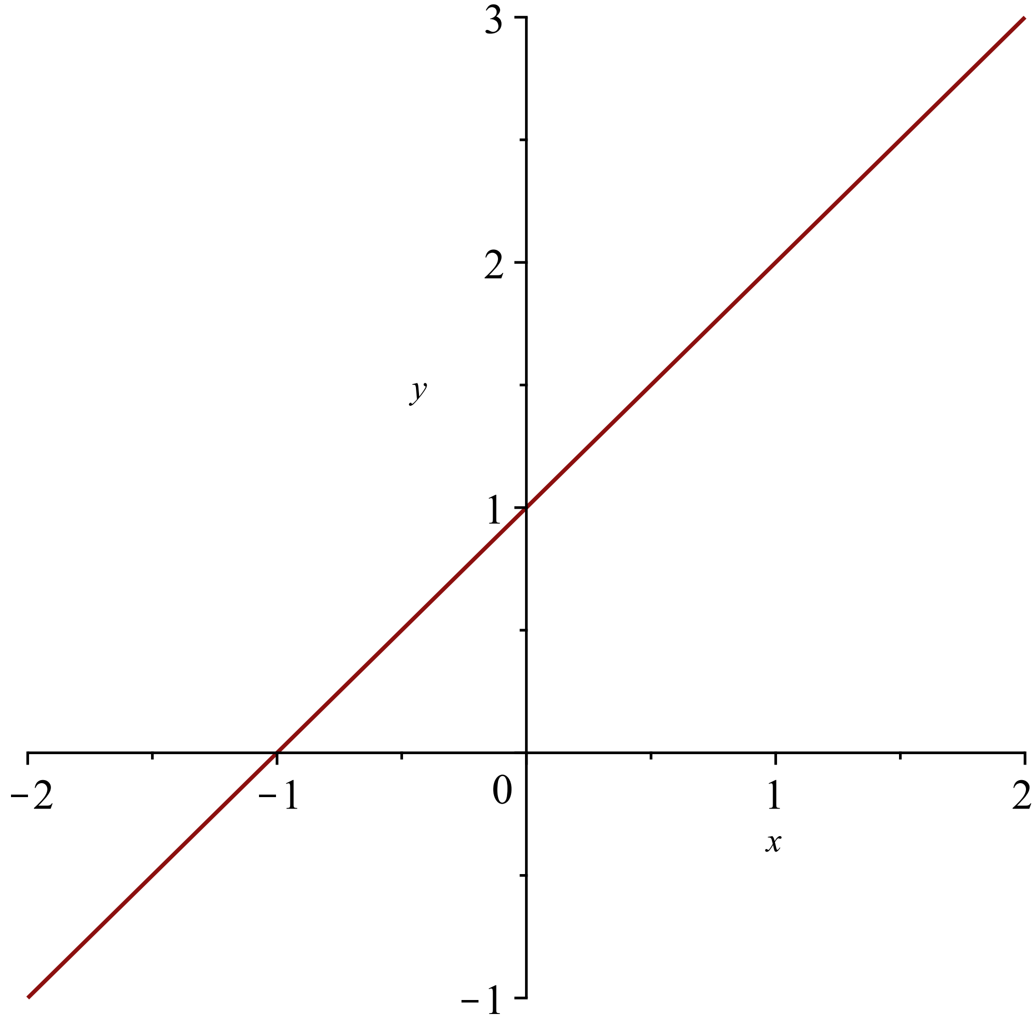
$$g := x \mapsto x + 1 \qquad \textbf{(7)}$$

> $plot(f(x), x = -2 .. 2, y = 0 .. 4);$

$plot(g(x), x = -2 .. 2, y = -1 .. 3);$



4
3
y
2
1
−2
−1
0
1
2
x

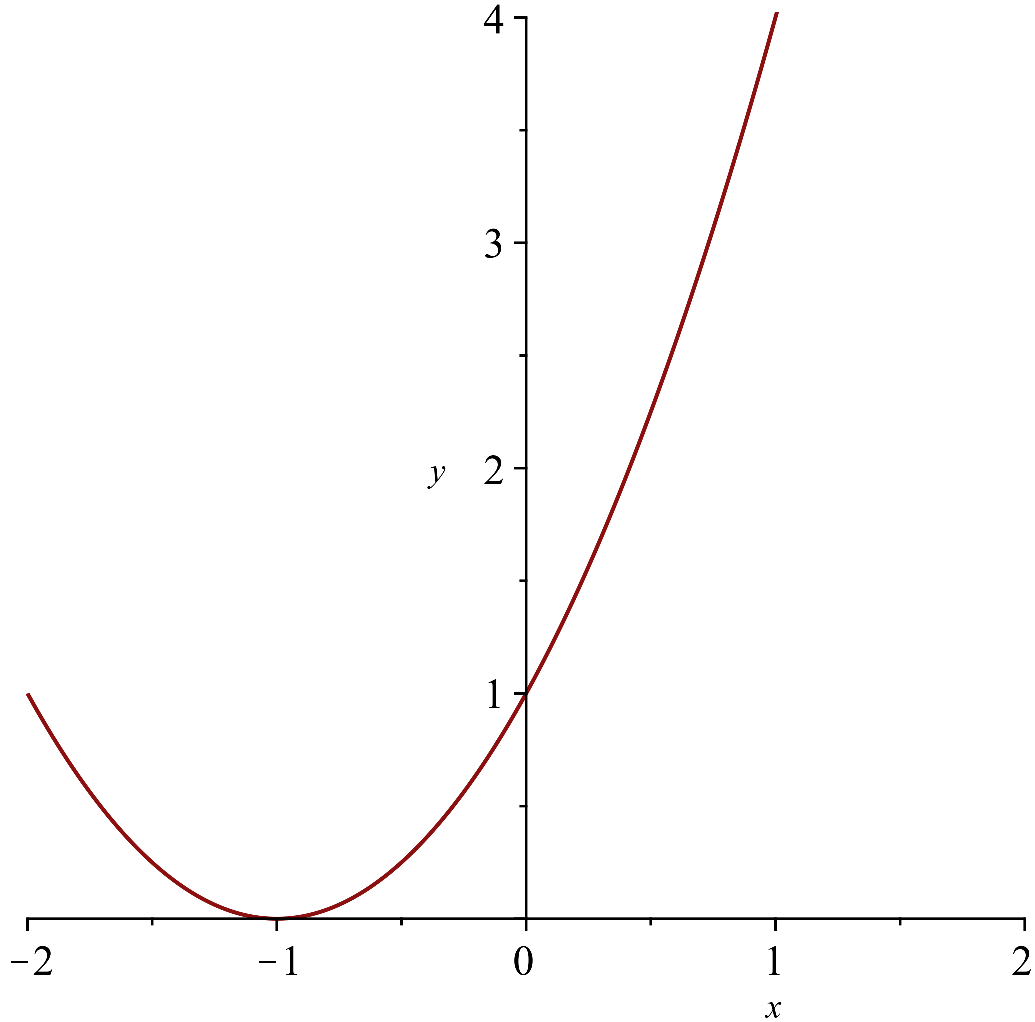


> $a := -2;$
> $b := 2;$
> $c := 0;$
> $d := 4;$

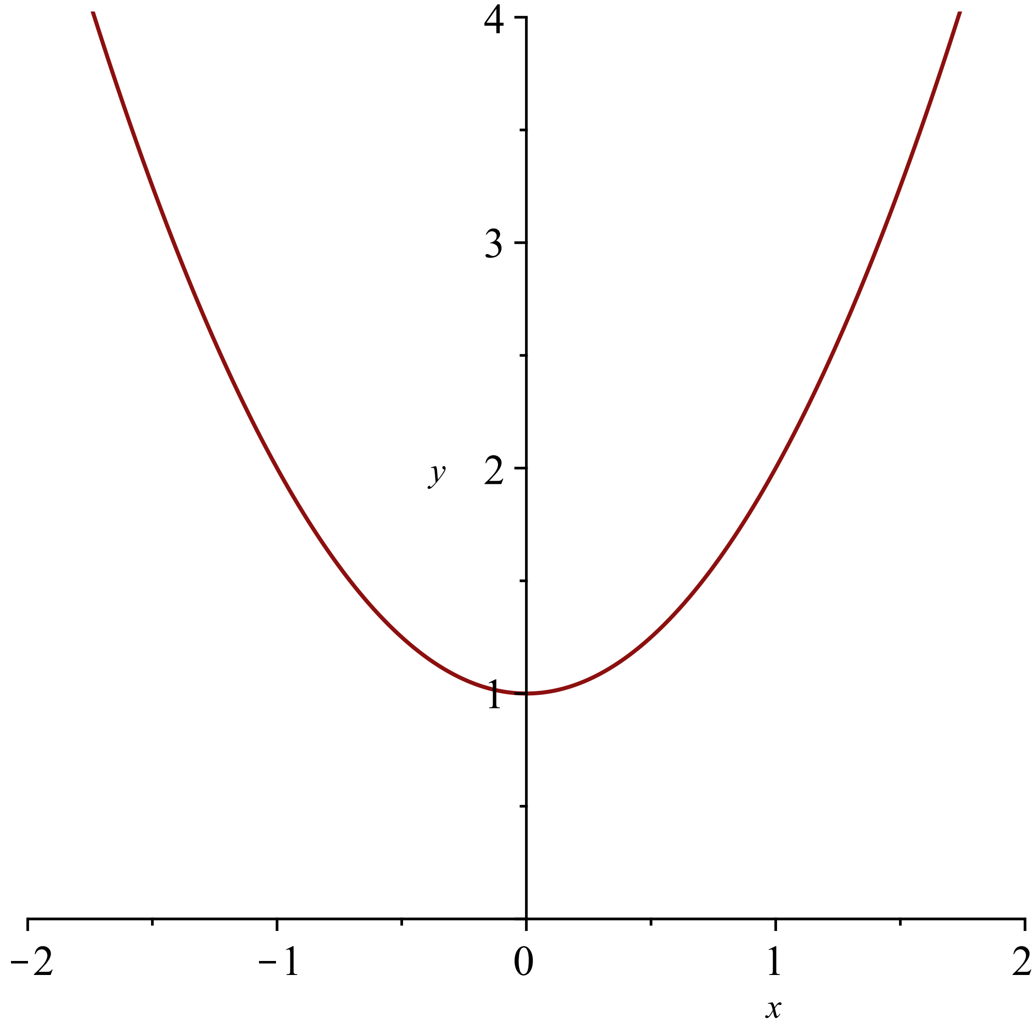
$$a := -2$$
$$b := 2$$
$$c := 0$$
$$d := 4 \tag{8}$$

> $f(g(x));$
> $plot(f(g(x)), x = a .. b, y = c .. d);$
> $g(f(x));$
> $plot(g(f(x)), x = a .. b, y = c .. d);$

$$(x + 1)^2$$



$x^2 + 1$



Komposition von quadratischer Funktion und zugehöroger inverser Funktion
(gilt für nichtnegative reelle Zahlen)

> *restart*;

> $f := x \rightarrow x^2$;
$g := x \rightarrow \text{sqrt}(x)$;

$$f := x \mapsto x^2$$

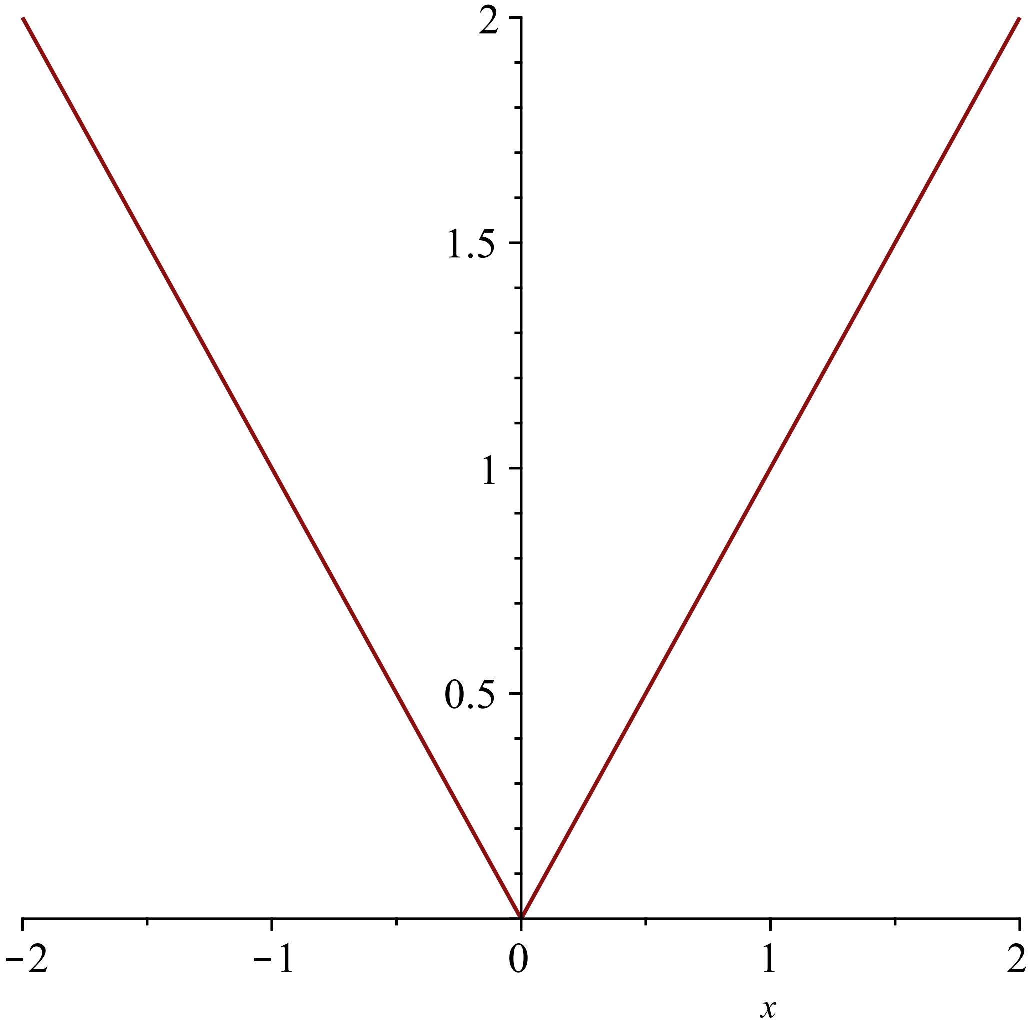
$$g := x \mapsto \sqrt{x} \tag{9}$$

> $f(g(x))$;
$g(f(x))$;

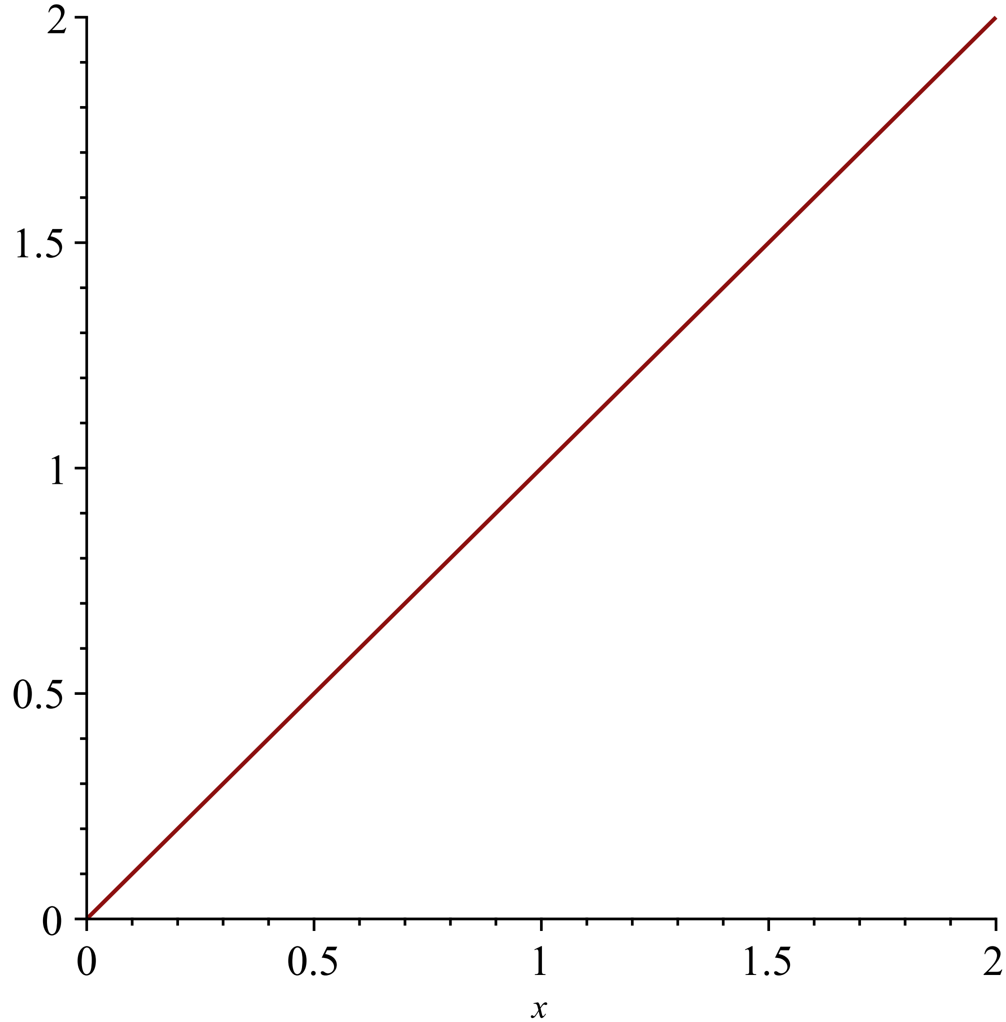
$$x$$

$$\sqrt{x^2} \tag{10}$$

> $plot(g(f(x)), x = -2 .. 2)$;



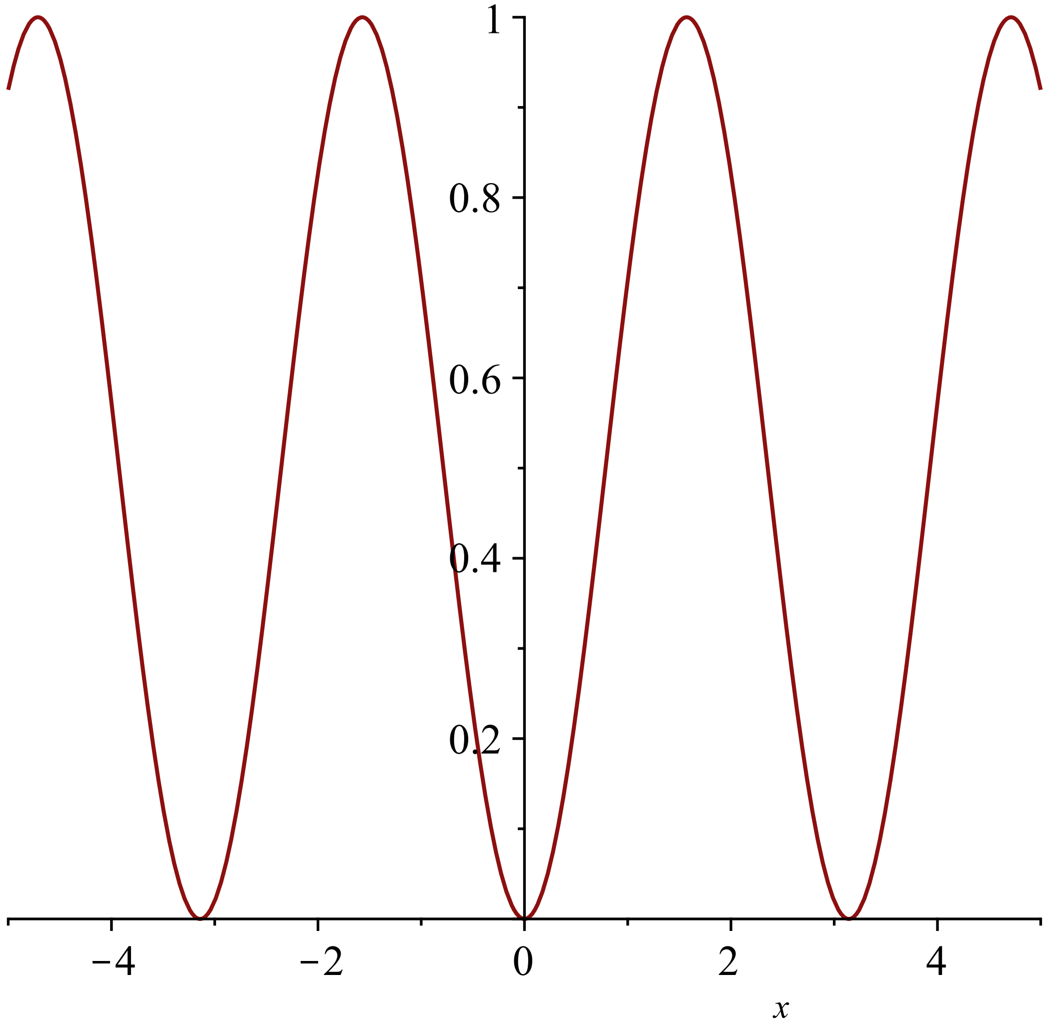
> $plot(g(f(x)), x = 0 .. 2);$



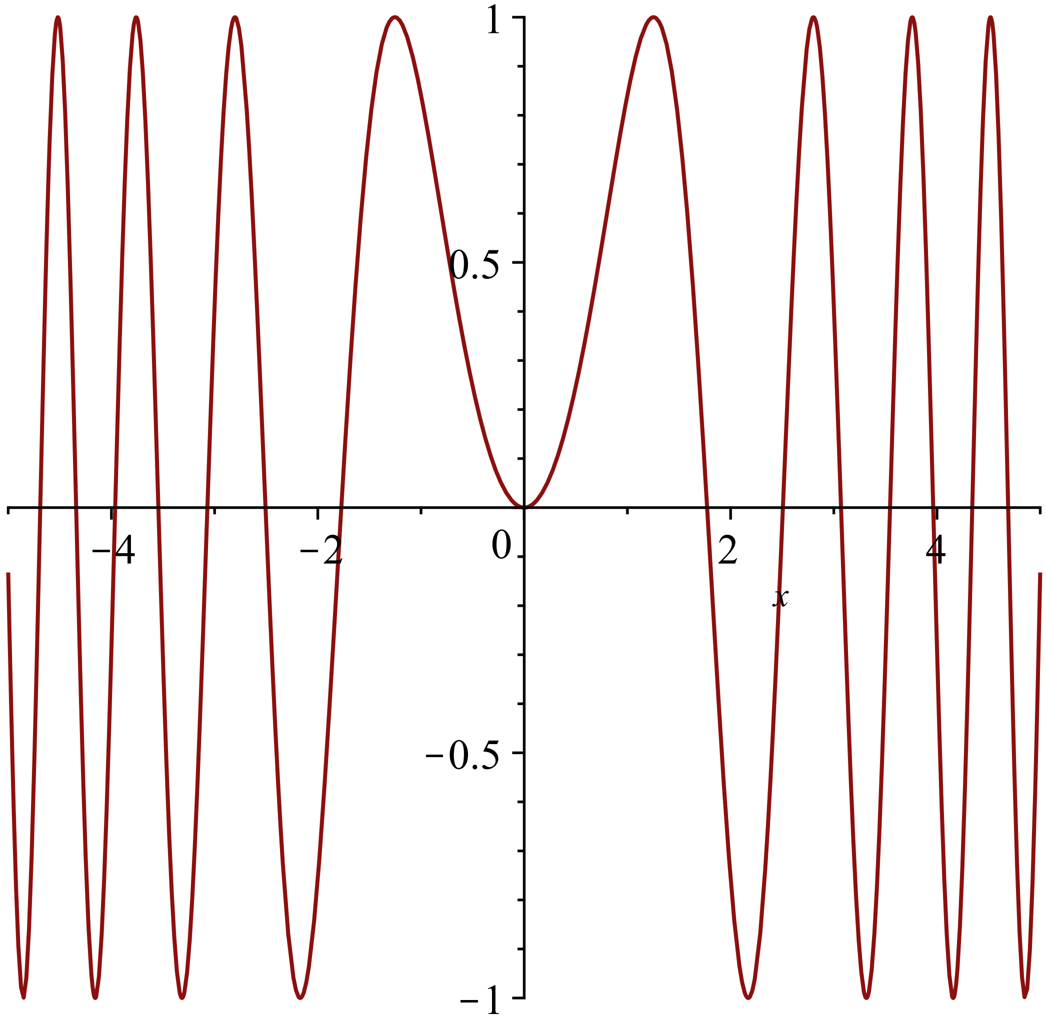
Komposition

> $f := x \to x^2;$
> $g := x \to \sin(x);$
> $f(g(x));$
> $plot(f(g(x)), x = -5..5);$
> $g(f(x));$
> $plot(g(f(x)), x = -5..5);$

$$f := x \mapsto x^2$$
$$g := x \mapsto \sin(x)$$
$$\sin(x)^2$$



$\sin(x^2)$



1
0.5
−0.5
−1
−4
−2
0
2
4
x

>

>