Schwingungsgleichung
Lineare Differentialgleichung für Auslenkung y(t)
(Reibungskonstante r > 0, Federkonstante k > 0, Masse m = 1)
Analytische Lösung mittels Eigenwertzerlegung und Variation-der-Konstanten Formel

```
> restart;
```

```
> Differentialgleichung := diff(y(t),t,t) + r*diff(y(t),t) + k*y(t) =
  f(t);
  Anfangsbedingungen := y(0)=y0,D(y)(0)=dy0;
```

$$Differentialgleichung := \frac{d^2}{dt^2}\, y(t) + r\left(\frac{d}{dt}\, y(t)\right) + k\, y(t) = f(t)$$

$$Anfangsbedingungen := y(0) = y0, \mathrm{D}(y)(0) = dy0 \qquad \textbf{(1)}$$

```
> Ab := y(0) = 1, D(y)(0) = 1;
```

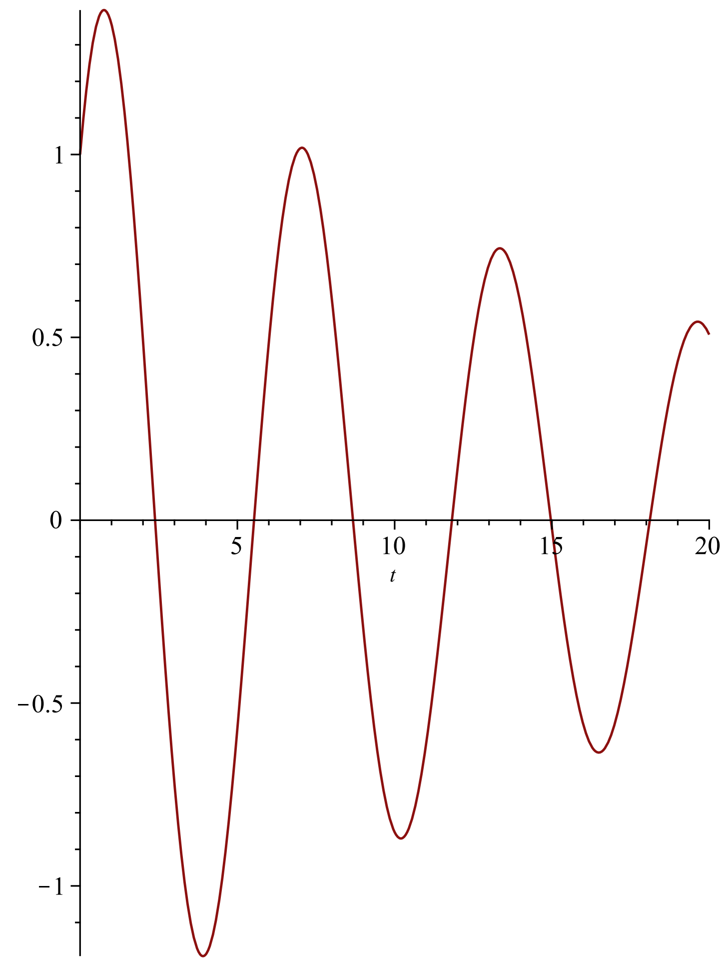
$$Ab := y(0) = 1, \mathrm{D}(y)(0) = 1 \qquad \textbf{(2)}$$

Periodischer Fall (r^2 - 4 k < 0)

```
> PeriodischerFall := subs({r = 1/10, k = 1, f(t) = 0},
  Differentialgleichung);
  Lsg := dsolve({PeriodischerFall,Ab}):
  Lsg := convert(Lsg,list)[2];
  plot(Lsg,t=0..20);
```

$$PeriodischerFall := \frac{d^2}{dt^2}\, y(t) + \frac{\frac{d}{dt}\, y(t)}{10} + y(t) = 0$$

$$Lsg := \frac{\sqrt{399}\, e^{-\frac{t}{20}} \sin\left(\frac{\sqrt{399}\, t}{20}\right)}{19} + e^{-\frac{t}{20}} \cos\left(\frac{\sqrt{399}\, t}{20}\right)$$



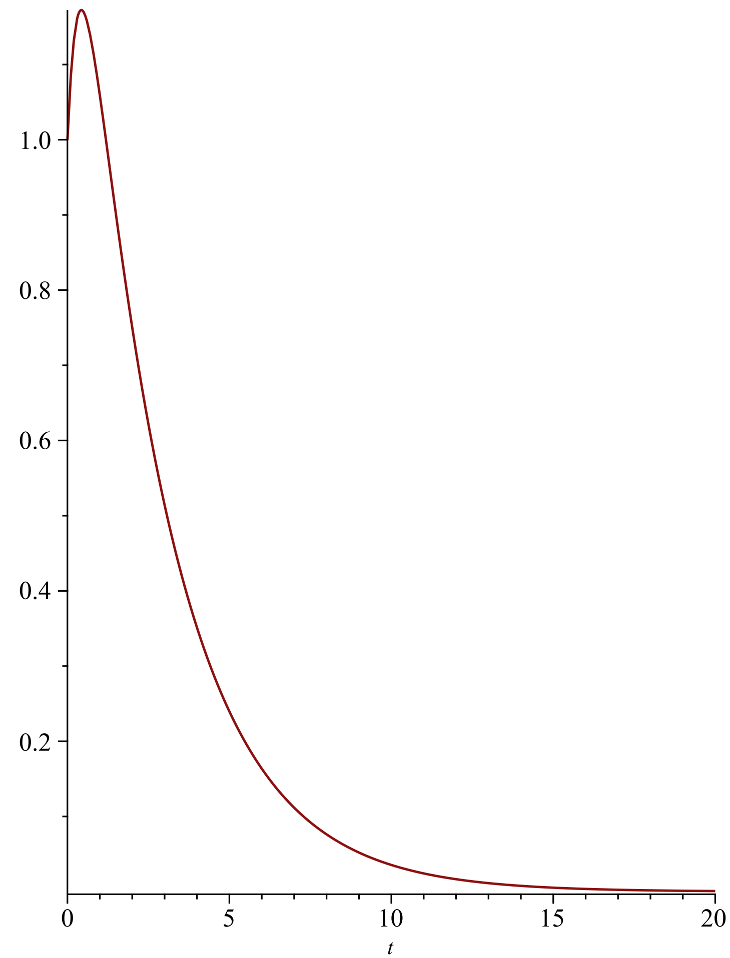
Aperiodischer Fall (r^2 - 4 k > 0)

```
> AperiodischerFall := subs({r = 3, k = 1, f(t) = 0},
  Differentialgleichung);
  Lsg := dsolve({AperiodischerFall,Ab}):
  Lsg := convert(Lsg,list)[2];
```

```
plot(Lsg,t=0..20);
```

$$AperiodischerFall := \frac{d^2}{dt^2}\,y(t) + 3\,\frac{d}{dt}\,y(t) + y(t) = 0$$

$$Lsg := \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) e^{\frac{(\sqrt{5}-3)\,t}{2}} + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) e^{-\frac{(3+\sqrt{5})\,t}{2}}$$



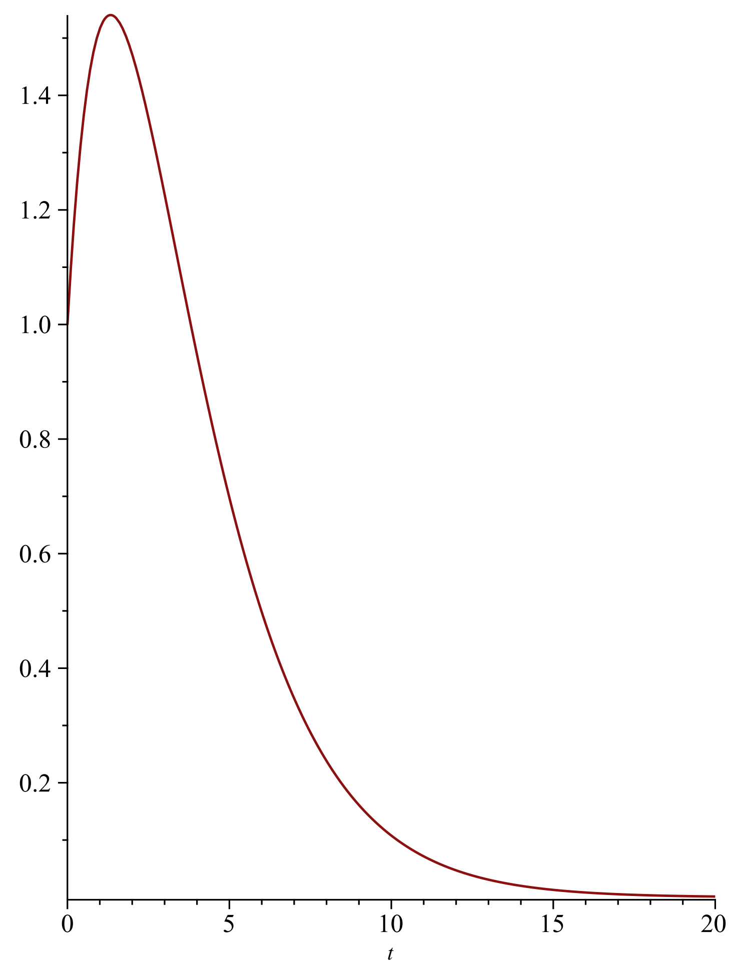
Aperiodischer Grenzfall (r^2 - 4 k = 0)

```
> AperiodischerGrenzfall := subs({r = 1, k = 1/4, f(t) = 0},
  Differentialgleichung);
  Lsg := dsolve({AperiodischerGrenzfall,Ab}):
  Lsg := convert(Lsg,list)[2];
```

```
plot(Lsg,t=0..20);
```

$$AperiodischerGrenzfall := \frac{d^2}{dt^2}\,y(t) + \frac{d}{dt}\,y(t) + \frac{y(t)}{4} = 0$$

$$Lsg := e^{-\frac{t}{2}} + \frac{3\,e^{-\frac{t}{2}}\,t}{2}$$



Zusätzliche äuflere Kraft (Sinus)

```
> AeussereKraft := subs({r = 1/100, k = 1, f(t) = sin(1.0001*t)},
  Differentialgleichung);
  Ab := y(0) = 1, D(y)(0) = 1;
  Lsg := dsolve({AeussereKraft,Ab}):
```

```
Lsg := evalf(convert(Lsg,list)[2]);
plot(Lsg,t=0..50);
```

$$AeussereKraft := \frac{d^2}{dt^2}\, y(t) + \frac{\frac{d}{dt}\, y(t)}{100} + y(t) = \sin(1.0001\, t)$$

$$Ab := y(0) = 1, \mathrm{D}(y)(0) = 1$$

$$Lsg := 3.503894375\, \mathrm{e}^{-0.005000000000\, t} \sin(0.9999875000\, t) + 100.9500250\, \mathrm{e}^{-0.005000000000\, t} \cos(0.9999875000\, t) - 1.998900560 \sin(1.000100000\, t) - 99.95002499 \cos(1.000100000\, t)$$